ELEN 4810 Homework 4

Analytical Questions

10.32 **(a)** Spectrograms (a) and (c) were computed with rectangular windows.

(b) The pairs (a & b) and (c & d) have approximately the same frequency resolution.

(c) Spectrogram (c) has the shortest time window. Spectrogram (a) clearly uses a longer window than (c); spectrogram (b) clearly uses a longer window than (d). So we need only compare (c) and (d). For (c), the mainlobe width is roughly 0.08 π. With a rectangular window, this corresponds to a length of $M = 49$. For (d), the mainlobe width is again roughly 0.08 π; however, this is a Hamming window, and so this corresponds to a length of roughly $M = 100$ samples.

(d) $\approx$ 350 samples. The window length is roughly the width of the transition regions that occur around 1,000 samples and around 2,000 samples. Answers that are close to 350 samples and/or exhibit correct reasoning are acceptable.

(e) As follows:

$$\cos(4,000\pi t + \phi_1) + \begin{cases} \cos(7,000\pi t + \phi_2) & 0 \le t \le 0.1s \\ 0 & 0.1s < t < 0.2s \\ \cos(5,000\pi t + \phi_3) & 0.2s \le t \le 0.3s \end{cases} \tag{1}$$

The phases $\phi_1 \ldots \phi_3$ cannot be determined from the given information. Answers noting some uncertainty in the amplitude will also be accepted as correct.

3.37. $X(z)$ has poles at $\frac{1}{2} + \frac{1}{2}j$, $\frac{1}{2} - \frac{1}{2}j$, $-\frac{3}{4}$, a zero at $z = 0$, and a zero of multiplicity two at $z = \infty$. Let $W(z) = \mathcal{Z}(x[n-3])$. Then $W(z) = z^{-3}X(z)$. $W(z)$ a zero of multiplicity five at $z = \infty$. It has poles at $\frac{1}{2} + \frac{1}{2}j$, $\frac{1}{2} - \frac{1}{2}j$, $-\frac{3}{4}$, and a pole of multiplicity two at $z = 0$. Using the time-reversal property of the $\mathcal{Z}$ transform, the poles and zeros of $Y(z)$ are the reciprocals of the poles and zeros of $W(z)$. In particular, $Y(z)$ has a zero of multiplicity 5 at $z = 0$, a pole of multiplicity two at $z = \infty$, and poles at $z = -\frac{4}{3}$, $z = \sqrt{2} + \sqrt{2}j$ and $z = \sqrt{2} - \sqrt{2}j$. The ROC of $Y(z)$ is $\{z \mid |z| < \frac{4}{3}\}$. The poles and zeros should be plotted.

3.40. **(a)** Notice that $X(z) = \frac{1}{1-z^{-1}}$. The $\mathcal{Z}$-transform of $(\frac{1}{2})^n u[n]$ is $\frac{1}{1-\frac{1}{2}z^{-1}}$. Using this and the delay property of the $\mathcal{Z}$ transform, we obtain that $Y(z) = \frac{1}{4}\frac{z}{1-\frac{1}{2}z^{-1}}$. We have

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{4}\frac{z(1-z^{-1})}{1-\frac{1}{2}z^{-1}} \tag{2}$$

$$= \frac{1}{4}\frac{z-1}{1-\frac{1}{2}z^{-1}}. \tag{3}$$

$H(z)$ has poles at $z = \frac{1}{2}$ and $z = \infty$. It has zeros at $z = 1$ and $z = 0$.

(b) By inspection, we can see that if we set $w[n] = (1/2)^n u[n]$, then

$$h[n] = \tfrac{1}{4}w[n+1] - \tfrac{1}{4}w[n] \tag{4}$$

$$= \tfrac{1}{4}(\tfrac{1}{2})^{n+1}u[n+1] - \tfrac{1}{4}(\tfrac{1}{2})^n u[n]. \tag{5}$$

(c) Yes, the impulse response is absolute summable, so the system is **stable**.

(d) No, $h[-1]$ is nonzero, so the system is **not causal**.

3.48. **(a)** The ROC of y is $\frac{1}{2} < |z| < 2$.

(b) Two sided.

(c) The ROC of x is $|z| > \frac{3}{4}$.

(d) Yes. The ROC extends outward, and so $x[n]$ is right sided. There is no pole at infinity (in fact, there is a zero at infinity) and so the Z transform power series contains no positive powers of z.

(e) There is a zero at $z = \infty$, and so $x[0] = 0$.

(f) $H(z)$ has **poles** at 2 and ∞, and **zeros** at 0 and $-3/4$. The ROC of H is $\{|z| < 2\}$.

(g) Yes. The ROC extends inward and so $h[n]$ is left sided. There is a zero at $n = 0$, and so the Z transform power series contains no negative powers of z.